Models for non-linear kinematic hardening at finite strain with application to metal forming processes

B. Svendsen[1], V. Levkovitch[2]

[1] Chair of Mechanics, University of Dortmund, 44227 Dortmund, Germany
[2] Laboratory V.23, Federal Institute for Materials Research and Testing, 12200 Berlin, Germany

In this work, an approach to the modeling of elastic and inelastic anisotropy in metallic materials at large deformation relevant to the simulation of metal forming processes is outlined. This approach is based on a thermodynamic, internal-variable-based formulation for such behaviour as based on mechanisms of induced anisotropy due to (i), texture development, and (ii), hardening behaviour, in particular kinematic hardening. In particular, the approach leads to specific forms for the constitutive relations and the reduction of the flow rule to an evolution relation for the plastic right Cauchy-Green deformation or its inverse for general anisotropic behaviour. Finally, issues pertaining to the implementation of the model and to the determination of its material parameters in the context of non-linear kinematic hardening are discussed.

1 Introduction

The quality of metal forming processes is determined among other things by how well building components achieve the form and size specified by the manufacturer. Among other things, this issue is influenced by the complex non-linear interplay between the process, material and structural parameters involved. In particular, the non-linear aspect of this interplay can lead to undesirable and to some degree unpredictable effects such as material and/or structural instability, or springback, leading to a poorer quality of the manufacturing process. The ability to predict, and so avoid, such effects is dependent among other things on how well the material and structural behaviour is modeled, as well as on how well the parameters of these models can be determined. Parameter identification depends in turn essentially on the ability of the experimentalist to carry out the corresponding experiments. In particular, in the inelastic context, it depends as well on being able to identify the transition from elastic to inelastic behaviour, *i.e.*, the point of inelastic yielding. Usually, departure from linearity (*e.g.*, [1]), or backward extrapolation (*e.g.*, [2]), or stress-strain offset (*e.g.*, [3,4]), methods are used to determine the initial as well as subsequent yield surfaces after loading has begun. These works demonstrate clearly how important the identification of the initial yield point is for the determination of the yield surface. Beyond this, the evolution of the yield surface with continued loading is also influenced significantly by preloading, by the type of loading, and by the loading path, investigated. Indeed, the yield surface may rotate, distort and/or translate during loading, leading in general to quite complex stress-strain behaviour.

Attempts tomodel anisotropic yield behaviour phenomenologically have focused on anisotropic (in particular orthotropic) generalizations of von Mises-type yield surface modeling(*e.g.*, see [4,5]). More generally, such induced anisotropic material behaviour in sheet metals, and more generally in metallic and polycrystalline materials, can be traced back to processes associated with texture and microstructural development (*e.g.*, [6]). A major simplicifation in the formulation of phenomenological material models for anisotropic material behaviour was achieved with the introduction of so-called structure tensors in [7] (see also [8-10]). To avoid confusion and to be more precise, the term "structure tensor" is used in this work solely to designate quantities which render the constitutive relations isotropic functions of their arguments. In order to model induced elastic and inelastic anisotropic material behaviour, such an approach has been extended more recently (*e.g.*, in the framework of the plastic spin, in [11]; in a thermodynamical setting, in [12,13]) to the modeling of deformation-like internal variables having to do with processes such as kinematic hardening or texture development. In particular, evolution of the deformation-like internal variables modeled in this fashion leads in general to induced orthotropic or yet more complex elastic and inelastic anisotropic material behaviour. One of the most important consequences of the modeling of internal variables as structure tensors is the reduction of the flow rule from nine to six dimensions. This is complementary to a previous result ([13]) showing that the stress measure thermodynamically conjugate to the rate of inelastic deformation is symmetric. In preparation for application to the modeling of induced elastic and inelastic anisotropy in sheet metal, this work outlines a simple application of the approach to the case of orthotropic metal plasticity with non-linear kinematic hardening.

2 Anisotropic viscoplasticity with non-linear isotropic and kinematic hardening

Most sheet metals are rate-dependent, even at room temperature. As such, a viscoplastic model for these is pursued here within the general framework proposed in [12-14]. In the current thermodynamic approach, this model is based on forms for the stored energy, overstress and evolution relations for the internal variables and structure tensors. The particular form of the model is formulated to be consistent with the micromechanical model in [15]. In the context of large deformation and evolving structure tensors describing induced elastic and inelastic anisotropy in metals, these potentials are formulated assuming small elastic strain. For the stored energy , we have the form

$$\psi = \tfrac{1}{2}\lambda(\mathbf{I}\cdot\ln\mathbf{V}_{\mathrm{E}})^2 + \mu(\ln\mathbf{V}_{\mathrm{E}}\cdot\ln\mathbf{V}_{\mathrm{E}}) + \tfrac{1}{2}\zeta\ln\mathbf{V}_{\mathrm{E}}\cdot\mathbf{A}\ln\mathbf{V}_{\mathrm{E}} + \tfrac{1}{2}\sum_{i=1}^{n} c_i\mathbf{A}_i\cdot\mathbf{A}_i + h(\varepsilon) \tag{1}$$

Here, λ and μ represent Lame's elasticity constants, $\ln\mathbf{V}_{\mathrm{E}}$ the second-order symmetric elastic left logarithmic stretch tensor, **A** the symmetric, traceless fourth-order elastic anisotropy tensor, and ζ an anisotropy constant. In addition, $\mathbf{A}_i$ is a strain measure for kinematic hardening in the material, and c_i the associated material parameter. As implied by (1), there are in general a number of such strain mechanisms contributing to kinematic hardening in real sheet metals. Finally, ε represents the accumulated equivalent inelastic strain in the material determining the process of isotropic hardening. Turning next to the overstress measure f , we work with the generalized von Mises form

$$f = \sqrt{\operatorname{dev}(\mathbf{K}-\mathbf{X})\cdot\mathbf{N}\operatorname{dev}(\mathbf{K}-\mathbf{X})} - \sigma_{\mathrm{F}} \tag{2}$$

for this in terms of the anisotropic inelastic flow tensor $\mathbf{N} = \operatorname{dev} + \eta\mathbf{A}$, the flow anisotropy strength parameter η, dev the deviatoric operator, and the stress measures

$$\begin{aligned}\mathbf{K} &= \psi_{,\ln \mathbf{V}_\mathrm{E}}\\ \mathbf{X} &= \sum_{i=1}^{n} \psi_{,\mathbf{A}_i} + 2\,\psi_{,\mathbf{A}_i}\,\mathbf{A}_i\\ \sigma_\mathrm{F} &= \psi_{,\varepsilon}\end{aligned} \tag{3}$$

represent the Kirchhoff stress $\mathbf{K}$, the back stress $\mathbf{X}$, and the yield stress σ_F, respectively. These are complemented by the implicit backward-Euler non-linear algorithmic forms

$$\begin{aligned}\ln \mathbf{V}_\mathrm{E} &= \ln \mathbf{V}_{\mathrm{E\,tr}} - \Delta\gamma\,\mathbf{N}\,\mathrm{dev}(\mathbf{K}-\mathbf{X})/\sqrt{\mathrm{dev}(\mathbf{K}-\mathbf{X})\cdot\mathbf{N}\,\mathrm{dev}(\mathbf{K}-\mathbf{X})}\\ \mathbf{A} &= \mathbf{A}_{tr} + \Delta\gamma\,\mathbf{G}(\ln \mathbf{V}_\mathrm{E},\mathbf{A})\\ \mathbf{A}_i &= \mathbf{A}_{i\,tr} + \Delta\gamma\, b_i(\ln \mathbf{V}_\mathrm{E} - \mathbf{A}_i)\\ \varepsilon &= \varepsilon_0 + \Delta\gamma\end{aligned} \tag{4}$$

of the evolution relations for the internal variables with respect to the current configuration. Here, $\ln \mathbf{V}_{\mathrm{E}tr}$, $\mathbf{A}_{\mathrm{i\,tr}}$ and $\mathbf{A}_{\mathrm{tr}}$ are so-called trial values of the corresponding tensors, and are known. Likewise, ε_0 is the known initial value of the accumulated equivalent inelastic strain ε. The form of the fourth-order-symmetric-traceless-tensor-valued function $\mathbf{G}$ is determined in [15] for face-centered-cubic systems with the help of micromechanics, crystal plasticity and texture analysis, reduced in the current work to the case of small elastic strain. Lastly,

$$\Delta\gamma = \Delta t \left\langle \tfrac{3}{2} f / \sigma_\mathrm{D} \right\rangle^{1/m} \tag{5}$$

is determined by the time-step magnitude Δt, the drag stress σ_D, and the strain-rate exponent m, where

$$\langle x \rangle = \tfrac{1}{2}\left(x + |x|\right) \tag{6}$$

represents the ramp function.

Finally, to demonstrate briefly the behaviour of this model, consider the standard case of cyclic simple shear. For simplicity, attention is restricted here to kinematic hardening only, *i.e.*, $\zeta = 0$ and $\eta = 0$ here. Further, a single kinematic hardening mechanism is assumed, such that $n = 1$ in (1) and $(3)_2$. The calculation is carried out for typical order-of-magnitude parameter values for certain metallic materials materials (*e.g.*, some kinds of steel), *i.e.*, $\lambda = 121154$ MPa, $\mu = 80769$ MPa, c = 2000 MPa, $b = 10$, and a constant yield stress of 500 MPa and $m = 1$. On this basis, results for the Kirchhoff stress $\mathbf{K}$ as a function of strain are presented for two cycles of simple shear of a single element in the 1-direction perpendicular to the 2-direction to 100% in Figure 1. Clearly, both a marked Bauschinger effect and strong saturation of kinematic hardening in the sense of Armstrong-Frederick are exhibited.

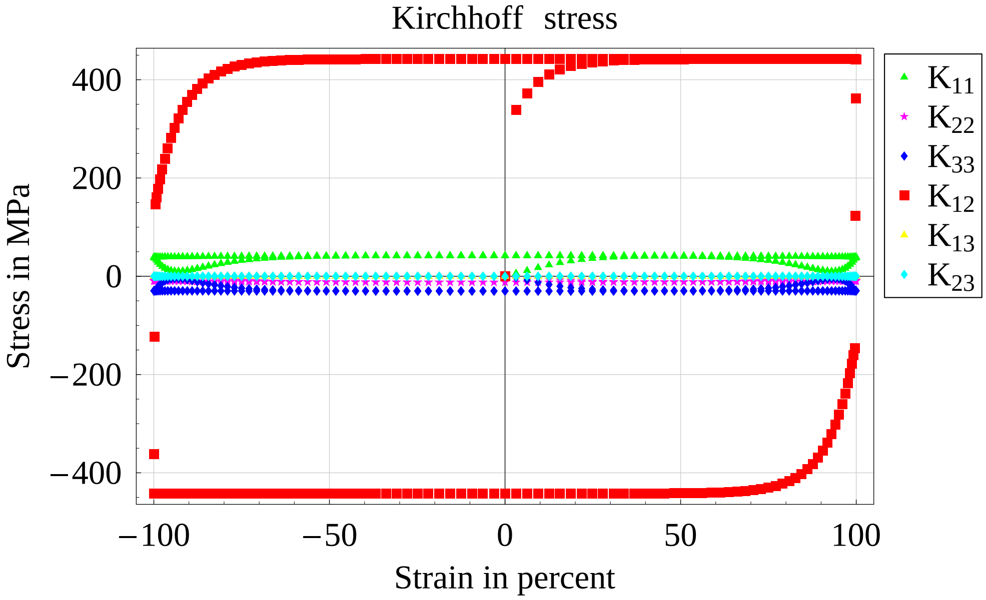

Figure 1: Simple shear in the 1-direction perpendicular to the 2-direction to 100%.

In the first phase of our project for SPP1138, which is beginning this fall, the material parameters of the above model will be determined with the help of (Myriachi und Teodosiu-type) shear and bending tests on sheet metals such as DP600. The corresponding identified material models will then be used to simulate free-bending and stip-tension-bending processes.

3 Schrifttum

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